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Strategic Firms and Endogenous Consumer Emulation*

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Abstract

We consider an equilibrium model of social learning with heterogeneous consumers and firms that act strategically. Consumers search for high qualities among a large set of firms, and can condition their choices on observed actions of other consumers. When they can recognize consumers who are more likely to have identified a high quality firm, uninformed individuals will optimally emulate those consumers. One group of consumers arises endogenously as “leaders” who are being emulated. Follow-on sales induce firms to give preferential treatment to these lead consumers, which reinforces their learning.

Keywords: Search, social learning, consumption signalling

JEL numbers: L15, D83

“One very clear impression I had of all the Beautiful People was their prudence. It may be that they paid for their own airline tickets but they paid for little else.”

James Brady, Press Secretary to Ronald Reagan
From *Superchic*, Little, Brown 1974

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1. Introduction

There is a large literature on social learning in which consumers make inferences about the quality of a good by observing what other consumers have done in the past. Banerjee (1992) and Bikhchandani, Hirshleifer and Welch (1992) analyze the case in which a sequence of identical individuals consume once and prices are fixed. Bose, Orosel, Ottaviani and Vesterland (1992a,b) extend the analysis to the case in which a monopoly seller facing a sequence of identical one-time buyers behaves strategically in setting his price.^{1,2} In this paper we integrate social learning into a more standard sequential search environment.³ We consider an infinite horizon problem in which a large number of firms with differing qualities face a large number of repeat buyers who make inferences about the quality of firms that other individuals have patronized in the past. Buyers are heterogeneous, and correct inference of an uninformed individual depends on which buyers are better informed, which in turn depends on the (endogenous) frequencies of purchase decisions.

We consider a market environment in which consumers are initially uninformed about firms' qualities, but will know a firm's quality perfectly after purchasing once. Some individuals consume more frequently than others, hence are on average better informed about quality. When uninformed consumers can identify those who are likely to purchase more frequently, they will optimally emulate a frequent purchaser.⁴ Because of the follow-on business of these frequent purchasers, they will (in equilibrium) be rewarded by firms they patronize. This will happen even though they do not pay more than others, and the price they pay does not cover the costs of the preferential treatment. Sales to these more frequent purchasers are in a sense "loss-leaders".

In our model consumers are heterogeneous with respect to income, which is assumed

¹Other work on social learning that is less related to the current paper includes Smallwood and Conlisk (1979) and Ellison and Fudenberg (1995) who consider information transmission of consumption choices or consumption outcomes, respectively, between boundedly rational consumers. Banerjee and Fudenberg (2004) extend this to word-of-mouth communication among rational agents.

²There is research in other fields on the degree to which consumer choice is influenced by other people. See Rogers (1995) for an overview over the marketing literature and for contributions to consumer research and reference groups see e.g. Bearden and Etzel (1982), Bearden et al. (1989) and Burnkrant and Cousineau (1975). In psychology see e.g. Cohen and Golden (1972) and Pincus and Waters (1977).

³Social learning has received little attention in sequential search. Ellison and Fudenberg (1995) analyze a model in which boundedly rational agents make repeated purchases, but due to the decision rule it can be reinterpreted as a model with finitely-lived agents who choose only at the beginning of their life based on a sample of other agents' utility in the previous period.

⁴The idea that quality might only be varified through purchases and subsequent consumption goes back at least to Nelson's (1970) concept of experience goods. He suggests that the pattern of an individual's repeated purchases might not be random, but incorporate the information of others, a process he terms guided sampling. We formalize the idea that this guiding might evolve endogenously with firms strategically engaged in the guided sampling.

observable. We assume that the good of unknown quality is normal, so the relatively wealthy consume more frequently, all else equal, and acquire information faster. Each consumer observes the choices of some other customers in the market. Individuals who have not found a product of acceptable quality have an incentive to buy from the same firms as the customers they observe, in the hope that those customers made informed decisions. When wealthier customers acquire information faster, other customers will follow their decisions rather than the decisions of poorer customers, if they observe both. Wealthier customers will then attract additional business to the firm they patronize. If the firm can enhance the buying experience of a customer by providing costly service, it will provide the service to their customers who purchase more frequently, both to prevent them from switching to a competitor or to induce them to consume more frequently. While typically wealthier customers receive preferential treatment, there may be additional equilibria in which special service is given to less affluent consumers. This arises because service reinforces the learning process, and exclusive service to the poorer consumers can lead them to purchase more frequently than the wealthy. This can only arise if service is sufficiently important.

In the analysis we focus on differences in income as the only source of heterogeneity between consumers. We will show that in a simple setting this translates into different opportunity costs in the search process. It will become clear in the analysis that the difference in the opportunity cost drives the results. Thus, whenever there is consumer heterogeneity that leads to one group consuming more frequently, uninformed consumers will follow the choices of members of the frequently-consuming group when possible. If the difference is observable, our model can be interpreted more broadly as an analysis of the interplay between consumer search and firm competition in a market with two-sided heterogeneity and service as the competitive strategic variable.⁵

We focus on income or wealth as the basis of heterogeneity for two reasons. First, for normal goods higher income is associated with higher consumption, which is the driving force of information acquisition in our model. Second, while income or wealth might be difficult to observe and usually have to be inferred by secondary characteristics, this might still be easier than to observe somebody's taste preferences. Observability is crucial in this setup since it is essential to a consumer's decision about whom to follow.⁶

The paper is structured as follows. The next section introduces the model. Section 3 derives the equilibrium. To reduce complexity of the analysis, we postpone interesting but straightforward discussions of the role of conspicuous consumption, visibility in the market and relative position as well as some robustness checks to section 4. Omitted

⁵In such a world service can in fact be reinterpreted as a price reduction, leading to price as the competitive variable. Note also that heterogeneity of opportunity costs in our model is identical to heterogeneity in valuations for the good.

⁶We discuss in the last section how conspicuous consumption may arise when wealth must be inferred rather than being directly observed.

proofs and derivations are provided in the appendix.

2. The model

This section sets out the model and the equilibrium concept, followed by a discussion of the main modelling choices. There is a countably infinite number of periods and a continuum I of consumers and a continuum J of firms. Consumers are heterogeneous with respect to their per-period income. Each consumer $i \in I$ has a type $\theta_i \in \{p, w\}$, indicating whether he is poor or wealthy. The proportion of wealthy consumers with income y_w is $\alpha \in (0, 1)$. The other consumers have income $y_p < y_w$.

Firms are infinitely lived and heterogeneous with respect to the quality $q \in \{q_l, q_h\}$, $q_l < q_h$, of the otherwise identical good that they produce. We denote the proportion of firms with quality q_h by $\lambda \in (0, 1)$.

Consumers' types are public information. A firm's type is initially known only to the firm, and is fully revealed to a consumer after consumption of the firm's output. Consumers die each period with probability $(1 - \delta)$; when a consumer dies, a new consumer of the same type is born; new agents know only the proportion of high quality firms.

The firm's problem

Each firm $j \in J$ supplies an indivisible good, the quality of which is exogenous and unchanging over time. The market price of the good, $P > 0$, is exogenously given and identical for all firms.⁷ Our focus is on firms' interest in attracting customers, and for simplicity we assume that the good can be produced costlessly. We assume that the firm chooses whether to provide service to a given customer, and denote the level of service by $s \in \{0, \bar{s}\}$, $\bar{s} > 0$, where 0 denotes no service. At the time of service provision, the customer is already locked in and cannot switch to a competitor for the current period. The cost $c(\bar{s})$ of providing service is c and is incurred in the period in which the service is provided. We assume $c > P$. There is no cost to the firm if service is not provided, i.e. $c(0) = 0$.

Firms can commit to any current customer to give service the next time he returns. More specifically, we model firm j 's choice $s_{j,i}^t$ in period t for consumer i as representing the firm's one-period-ahead service commitment. $s_{j,i}^t$ is the promise to provide this service level in the first period $\tau > t$ that the customer returns. First period service is zero.

Let $I_j^t(\theta)$ be the set of consumers of type θ consuming products of firm j in period t , and $\hat{s}_{j,i}^t$ the service that firm j actually provides to consumer i in period t . Then firm

⁷We argue in the discussion section that the price could be endogenized without qualitatively affecting our results.

j 's per period profit is

$$\pi_j^t = \int_{i \in I_j^t(w)} P - c(\hat{s}_{j,i}^t) di + \int_{i \in I_j^t(p)} P - c(\hat{s}_{j,i}^t) di.$$

Firms maximize the discounted present value of per period profits, $\sum_{t=0}^{\infty} \beta^t E \pi_j^t$.

The cost of the provision of service is shown in the per period profit expression above while the benefits are indirect. A firm that promises service to an individual consumer may deter the consumer from switching to a competitor or may increase the frequency with which he patronizes the firm. Furthermore, the consumer's choice may affect the future choices of others. These (potential) benefits to a firm that provides service are reflected in the size of the set of consumers who consume at the firm in the future. As a tie-breaking rule we assume that firms offer service when indifferent.

The consumer's problem

Consumers are heterogeneous with respect to income, but all can afford the product. That is, $y_w > y_p > P$. Income is non-storable. In each period $t \in T$ each consumer $i \in I$ has the two choices: Enter the market or not, and if entering the market, from which firm to consume. If he does not enter, that is, does not purchase the good, he spends his income on a numeraire good. Since income is non-storable and the price of the numeraire is normalized to one, the consumer obtains y units of the numeraire in the case that he does not consume in the market, and $y - P$ units if he does.

At the beginning of each period, before the consumption decision is made, a taste shock ρ is realized for each consumer that affects the degree to which he enjoys consuming the indivisible good in that period. We assume this shock is an i.i.d. draw from distribution F with density f and full support on $[\underline{\rho}, \bar{\rho}]$, where $-\infty \leq \underline{\rho} < \bar{\rho} \leq \infty$. If the consumer decides to enter the market and consume from firm j in period t , his utility in that period is

$$U^t = q_j + \hat{s}_j^t + \rho^t + u(y - P),$$

where q_j is the quality of firm j , \hat{s}_j^t is the service that he receives and ρ^t is the current period taste shock. $u(\cdot)$ denotes the utility derived from the numeraire, which is assumed to be increasing and strictly concave.

If the consumer is uninformed and chooses a firm randomly, his expected utility is

$$EU^t = E_j[q_j] + \rho^t + u(y - P).$$

If the consumer decides not to consume, his utility for that period is $U^t = u(y)$. Consumers maximize the expected discounted utility $\sum_{t=0}^{\infty} \delta^t EU^t$, where $\delta \in (0, 1)$ is the probability of survival.

We assume that observing other consumers' behavior partially substitutes for an individual's initial lack of information about product qualities. After the first time

a consumer purchases the indivisible good, he can costlessly observe at which firm a random wealthy consumer and a random poor consumer consumed in the previous period.⁸ This implies that only players who participated in the market in the previous period are observable. For ease of exposition we assume that if a consumer is indifferent between following other participants' choices observed at different periods, he follows the most recent observation.

Equilibrium

In the spirit of Markov perfection we are interested in equilibria in which firms and consumers base their decisions only on information that is relevant for their future payoffs. We also restrict attention to anonymous strategies which condition only on the type (i.e., wealthy or poor) but not on the name of other players. For firms, the minimal payoff relevant information is the type of the consumer. We therefore consider equilibrium strategies $s(\theta)$ in which the service commitment of a firm is a function of the consumer's type. We will consider symmetric pure strategies, that is, $s(\theta)$ is deterministic and the same for firms with the same type. We denote by $s_l(\cdot)$ the strategy for low and by $s_h(\cdot)$ the strategy for high quality firms.

For a consumer, the relevant information is the combination of quality and service he can obtain; the name of the firm(s) from which he can receive that combination is not important. Consequently, a consumer conditions his actions on $D \subseteq \{q_l, q_h\} \times \{0, \bar{s}\}$, where D denotes the various quality-service combinations he has experienced. If the consumer has not yet purchased, $D = \emptyset$. A strategy for a consumer of type $\theta \in \{p, w\}$ is then a tuple $(\hat{p}^\theta(D), \sigma^\theta(D))$ for all D . The term $\hat{p}^\theta(D)$ denotes a reservation value for the taste shock: If the taste shock is above $\hat{p}^\theta(D)$, the consumer buys the product, otherwise he does not.⁹ If he chooses to buy, $\sigma^\theta(D)$ specifies the choice of firm from which he purchases the good. If $D \neq \emptyset$, then the consumer can return to a firm with quality-service combination $(q, s) \in D$, he can follow the choice of either a wealthy or a poor player observed in the previous period, or he can search randomly for a new firm. If $D = \emptyset$ only the last option is available, as by assumption the consumer must search on his own in the first period of consumption.

Let $n^{\theta,t}(D)$ denote the measure of type θ consumers with information D at time t , where the law of motion is determined by the strategies of consumers and firms. With this we can define a stationary equilibrium encompassing stationary distributions (1),

⁸In section 4.2 we discuss alternative signal technologies. Observing more than one player of any type does not alter any results. Observing only a random selection of N players' choices each period should not alter any qualitative results as long as N is sufficiently large.

The specification that newborn players in the market have no information about other players' choices prevents a situation in which everybody is following other market participants and no player is searching randomly. The specification that only players who have consumed at least once in the market can observe other players simplifies the analysis.

⁹In the appendix we show that reservation strategies are indeed optimal in our environment.

consumer optimality (2) and firm optimality (3):

Definition 1 (Stationary Equilibrium) *A stationary equilibrium is a tuple $S = (s_l, s_h, (\hat{\rho}^w, \sigma^w), (\hat{\rho}^p, \sigma^p), n^w, n^p)$ such that*

1. $n^{\theta,t}(D) = n^\theta(D) \forall t \forall D \forall \theta$.
2. *For each consumer of type θ , strategy $(\hat{\rho}^\theta, \sigma^\theta)$ is optimal in the continuation game for all D , when the consumer takes as given the strategies and fractions of the other players as summarized in S .*
3. *For each high (low) quality firm s_h (s_l) is optimal given S .*

Discussion of modelling assumptions

We will briefly discuss the modelling choices. A discussion of alternative assumptions and their impact on our results is relegated to section 4.2.

Firms. We allow one-period-ahead commitment in order to eliminate implausible equilibria. In particular, without commitment there is always an equilibrium in which a firm does not provide service because the firm cannot convince the customer that he will also get service in the future. In that case, giving service today is costly while it does not change the future behavior of the customer, which implies that it is unprofitable to provide service today. Therefore, even if the firm would like to give service to deter the customer from switching to a competitor or to encourage the customer to consume more frequently, it omits service because it cannot affect the customer's belief about future service through current period action. This arises no matter how profitable the business with this customer is. We restrict attention to time-independent firm strategies to focus on the effects of information transmission and to abstract from repeated game effects. Without commitment, this implicitly restricts the beliefs of the consumer to be unaffected by current period actions, rendering current period service an ineffective tool to change consumer behavior. Limited commitment power, i.e., commitment only for the next time in which the customer returns, provides a tool by which a customer's beliefs about future service can be altered. This allows firms to provide service if the business with the consumer is sufficiently profitable. In the last section we argue that the equilibrium with commitment will also be an equilibrium of the game without commitment when we allow for non-Markovian strategies.

The assumption that no service is given during the first period of consumption at a firm is also due to our restriction to Markovian strategies. If firms have a choice to give more service than promised, they would not do so since they cannot influence the consumer's future behavior. Since there is no commitment to service in the first period, they would not provide any service.

Taking prices as exogenously given allows us to focus on private information that is not fully revealed through prices. That prices do not reveal all relevant information about products is widely accepted. Strong restrictions on pricing behavior are therefore common in models of this sort to preclude revelation of too much information (see, for example, Wolinsky (1990)). On the other hand we can, with slight modification, interpret the service as a price reduction. In this sense we do allow for price competition. We discuss the possibility of price competition further in section 4.2.

Consumers. The combination of the numeraire good as an alternative to market consumption and the taste shock together explicitly capture the idea that the good under consideration is a normal good. The opportunity cost of going into the market is

$$u_0 := u(y) - u(y - P),$$

that is, the opportunity cost of foregone consumption of the numeraire good. Denote this opportunity cost u_0^w for the wealthy and u_0^p for the poor. The strict concavity of $u(\cdot)$ then implies that $u_0^w < u_0^p$, that is, the wealthy have a lower opportunity cost of consumption than the poor.¹⁰ Without service the wealthy will therefore consume at lower values of the taste shock than the poor. Thus, on average the wealthy consume more often, which establishes our version of the normal goods assumption.¹¹

The taste shock also allows firms to encourage the customer to consume more frequently. The timing of when to consume in the market is not exogenously fixed, but rather depends on the current period taste shock and the utility of consumption. By promising service, the firms can raise the utility of consumption and can thus encourage a consumer to consume more frequently.

Our results are most interesting when $c > P$. In this case no consumer will receive service only because of his own consumption at a firm. Firms will only provide service because a consumer brings in additional customers who follow his lead. Consequently, this case clearly highlights the effects of information transmission in the market.

Equilibrium. The equilibrium concept requires optimality in the continuation after a deviation to ensure that the firms' service promises affect behavior.

¹⁰Players get in each period $y - P$ units of the numeraire for sure, independent of their current period choice. Therefore wealthy players consumption-independent level of the numeraire is higher. Only the additional amount that they might get, i.e. their opportunity cost, is lower. The term $u(y - P)$ in the utility function will be dropped for all subsequent calculations as it only reflects a constant.

¹¹Heterogeneity in the opportunity costs of consumption (rather than the heterogeneity in terms of income) can be taken as primitive to allow for more general interpretations of the model. See section 4.1 for a discussion.

3. Optimal Behavior

As a first step to characterizing the stationary equilibria of this game, in the following subsection we describe the optimal consumption behavior of a single consumer. Toward this end, we consider a partial problem in which the service provision by firms and the information in the market is exogenous. In the next subsection we endogenize the market information, still under the assumption that firms service provision is exogenously given. The subsequent subsection analyzes the behavior of the firms. Finally, we integrate the parts in the analysis of the equilibria.

3.1. Consumer Search

Consider a consumer with opportunity cost u_0 who has entered the market at least once, that is, he will observe other participants in the market. Suppose all high quality firms offer identical service s (either \bar{s} or zero) to this consumer in every period, and low quality firms do not offer more service than high quality firms.¹² If the consumer has not found a high quality firm and chooses to purchase from a firm that he hasn't previously frequented, there is a probability which we denote γ that this firm will be of high quality. We take for now as exogenous the process by which this consumer chooses a new firm, and hence γ .

We are interested in the optimal decision rule of the consumer. The problem is a standard search problem with one exception – the consumer does not search in every period. Rather, the choice to enter the market is endogenous and depends on the taste shock, where high taste shocks imply that he enjoys consumption of the indivisible good relatively more than when the taste shock is low. It also depends on the qualities and service promises of the firms he has encountered, as well as on his beliefs about the service he will be promised by firms he has not yet encountered. The decisions about how frequently to search and how to choose firms given the frequency of search are interlinked: a higher frequency of search effectively implies a higher discount factor in the choice of firms. Higher quality or higher service imply a higher value of consumption, thus increasing the frequency of consumption.

Using standard techniques, appropriately modified to this setting, we derive cutoff levels $\hat{\rho}_l$ and $\hat{\rho}_h$ for the taste shock as functions of the quality of the best firm encountered so far, for which the consumer is indifferent between consuming in the market and not consuming. When the consumer has only experienced low quality firms, he will enter the market only if his taste shock ρ exceeds the cutoff $\hat{\rho}_l$, and his frequency of consumption is thus $(1 - F(\hat{\rho}_l))$. If the customer has experienced at least one high quality firm, his

¹²We will later show that it is always profitable for a high-quality firm to provide service if it is profitable for a low-quality firm to provide service. Therefore this specification will be the relevant case.

relevant cutoff is $\hat{\rho}_h$ and his frequency becomes $(1 - F(\hat{\rho}_h))$. Both $\hat{\rho}_l$ and $\hat{\rho}_h$ depend on the service strategies of the firms.

To ensure that a cutoff exists that leads to this indifference, we make the following assumption on the support of the taste shock that we will retain throughout: $(\underline{\rho}, \bar{\rho}) \supset [u_0^w - q_h - \bar{s}, u_0^p - q_l]$. This implies that $\underline{\rho} + q_h + \bar{s} < u_0^w$, i.e., even in the most advantageous situation of high quality and high service, there are taste shocks sufficiently low such that not consuming is preferable. Similarly, it implies that $q_l + \bar{\rho} > u_0^p$, so that even in the most disadvantageous situation some taste shocks still induce the consumer to enter the market. Requiring the support to be sufficiently large avoids the discussion of boundary cases. Note that we consider different ranges for \bar{s} in some of our statements. If a range conflicts the first inequality, it is vacuous by assumption. Yet for any shock distribution that has unbounded lower support, i.e. $\underline{\rho} = -\infty$, the inequality above does not restrict \bar{s} in any way and all ranges that we consider are possible.

Before stating the results on the consumer's search behavior, it should be noted that the consumer might not search for high quality firms if all firms offer service since the consumer does not receive service in his first period of consumption at a firm. If he returns to a low quality firm, he will receive service immediately, while he will not receive any service if he continues searching for a high quality firm until he consumes at least twice at such a firm. However, if service is not too important, or the consumer is sufficiently patient, he will search for high quality firms rather than remaining with a low quality firm. We summarize this in the following lemma, which is proven in the appendix.

Lemma 1 *If all firms offer service, there exists $\underline{\delta} \in [0, 1)$ such that for $\delta \geq \underline{\delta}$ the consumer searches for high quality. If $\bar{s} < \gamma(q_h - q_l)$, consumers always search for high quality firms, i.e. $\underline{\delta} = 0$.*

The following lemma describes the optimal service strategy for the consumer. Recall that s denotes the service promised by high quality firms to this consumer, which is assumed weakly larger than the service promised by low quality firms. Let

$$\hat{\rho}_l = u_0 - E_\gamma(q) - \frac{\delta\gamma}{1-\delta} \int_{u_0 - q_h - s}^{\hat{\rho}_l} [1 - F(\rho)] d\rho \quad (3.1)$$

where $E_\gamma(q) = \gamma q_l + (1 - \gamma)q_h$. Let the state variable q be the best quality yet encountered. Then we obtain:

Lemma 2 *If $\delta \in (\underline{\delta}, 1)$ and high quality firms offer weakly higher service, then the consumer's optimal decision rule has the following structure:*

If $q = q_l$, sample a new firm if current period shock $\rho \geq \hat{\rho}_l$, otherwise don't consume. If $q = q_h$, then return to the firm with high quality if the current period shock $\rho \geq \hat{\rho}_h = u_0 - q_h - s$, otherwise don't consume.

Intuitively these cutoffs are easy to understand. At high qualities the consumer either gets u_0 or $q_h + s + \rho$, and he chooses the higher. At low qualities the trade-off is similar, except that consumption yields only the average quality $E_\gamma(q)$. However, at low qualities there is a future benefit of today's experimentation: Finding a high quality firm yields benefits in future periods of consumption and, thereby, also induces a higher frequency of consumption in the future. These benefits are reflected in the increment in consumption frequency in the last term on the right hand side of (3.1).

The primary interest in the lemma stems from its implications for the behavior of wealthy consumers, i.e., those with the lower opportunity cost, relative to the behavior of poorer consumers. To compare the two groups, consider a setting where high quality firms promise service $s_h(p)$ to poor consumers and $s_h(w)$ to wealthy consumers, and low quality firms offer at most this level of service: $s_h(\cdot) \geq s_l(\cdot)$. Let $\hat{\rho}_l^p$ and $\hat{\rho}_h^p$ be the threshold levels for a poor consumer, and $\hat{\rho}_l^w$ and $\hat{\rho}_h^w$ be the threshold levels for a wealthy consumer. The following lemma compares these cutoff values:

Lemma 3 Let $\delta \in (\underline{\delta}, 1)$ and $s_h(\cdot) \geq s_l(\cdot)$.

- 1) If $s_h(w) \geq s_h(p)$, then the wealthy consume more frequently than the poor: $\hat{\rho}_l^w \leq \hat{\rho}_l^p$ and $\hat{\rho}_h^w \leq \hat{\rho}_h^p$.
- 2 a) If $s_h(p) - s_h(w) = \bar{s} < u_0^p - u_0^w$, still $\hat{\rho}_l^w < \hat{\rho}_l^p$ and $\hat{\rho}_h^w < \hat{\rho}_h^p$.
- 2 b) If $s_h(p) - s_h(w) = \bar{s} > u_0^p - u_0^w$, then $\hat{\rho}_h^w > \hat{\rho}_h^p$.
There exists unique $\xi_\gamma > u_0^p - u_0^w$ such that $\hat{\rho}_l^w < \hat{\rho}_l^p$ if $\bar{s} < \xi_\gamma$ and $\hat{\rho}_l^w > \hat{\rho}_l^p$ if $\bar{s} > \xi_\gamma$.

Proof: The result for the cutoff $\hat{\rho}_h$ follows directly from $\hat{\rho}_h^\theta = u_0^\theta - q_h - s_h(\theta)$, $\theta \in \{p, w\}$. For $\hat{\rho}_l$, rewrite (3.1) as $\hat{\rho}_l^\theta - u_0^\theta + E_\gamma(q) = -\frac{\delta\gamma}{1-\delta} \int_{u_0^\theta - q_h - s_h(\theta)}^{\hat{\rho}_l^\theta} [1 - F(\rho)] d\rho$ and observe that the left hand side is increasing in $\hat{\rho}_l^\theta$ and decreasing in u_0^θ and the right hand side is decreasing in $\hat{\rho}_l^\theta$ and increasing in $u_0^\theta - s_h(\theta)$. For 1) and 2a) the wealthy have lower u_0^θ and $u_0^\theta - s_h(\theta)$, therefore their cutoff $\hat{\rho}_l^w$ must be lower for the equality to hold. For 2b), if $u_0^p - s_h(p) \approx u_0^w - s_h(w)$, then $\hat{\rho}_l^w < \hat{\rho}_l^p$ since $u_0^w < u_0^p$. Since $\hat{\rho}_l^p$ is by (3.1) strictly increasing and unbounded in \bar{s} when $s_h(p) - s_h(w) = \bar{s}$ but $\hat{\rho}_l^h$ is constant, there exists a unique ξ_γ such that $\hat{\rho}_l^w = \hat{\rho}_l^p$ if $\bar{s} = \xi_\gamma$. *Q.E.D.*

If wealthy and poor consumers are treated equally by firms, this result simply restates our formulation of the normal goods assumption. The wealthy consume more frequently both in the search phase and after they have found a high quality firm. The

explicit formulation allows us to compare the frequency of consumption even in the cases where consumers are treated differently by firms. As long as the service benefit does not outweigh the differences in the opportunity costs of consumption, wealthy consumers still consume more frequently even if poor consumers are treated preferentially. If the impact of service outweighs the difference in the opportunity costs, the frequency of consumption at high quality firms becomes larger for the poor than for the wealthy. This does not necessarily translate into a higher frequency of consumption at low quality firms. As long as consumers are searching they do not obtain service, and without service the wealthy have a stronger incentive to consume. Only if the service at high quality firms is sufficiently attractive, poor consumers search more frequently for high qualities than the wealthy. Otherwise it still means that wealthy consumers find high quality firms relatively faster, even if service outweighs the exogenous difference in opportunity costs.

We turn next to endogenizing the probability γ .

3.2. Consumers' stationary behavior

In equilibrium, both high and low quality firms decide whether to provide service to each of the two types of consumers. Before examining the full model, we take the firms' choices regarding service as fixed and examine consumers' behavior in steady state as they choose optimally, given firms' choices. We will again consider the case where all high quality firms offer weakly higher service than low quality firms.

In this case all consumers, wealthy and poor, will sample in a way that gives the highest probability of identifying a high quality firm. Thus, both the uninformed wealthy consumers and the uninformed poor follow the same group. The probability of drawing a high quality firm, γ , when following consumers of this group (which we will call "leaders") is now endogenous. γ depends on the particular stationary equilibrium we are looking at.

We note that following *any* consumer, wealthy or poor, is preferable to searching randomly. At worst, that consumer who is followed has not found a high quality firm yet, in which case the firm he or she purchased from is as likely to be high quality as a randomly sampled firm. In addition, there is positive probability that the consumer who is being mimicked has found a high quality firm and purchases only from that firm. Hence, it is strictly better to follow *any* other consumer than to sample randomly; thus, in any equilibrium $\gamma \geq \lambda$. Due to this inequality $\underline{\gamma}$ in lemma 1 can be established independent of the exact value of γ , and long-lived consumers indeed search for high qualities.

An individual who follows the leaders sees only those who have consumed in the previous period. The probability that this individual will find a high quality firm is the proportion γ of the leaders who have identified a high quality firm and who consumed

in the previous period. When the wealthy identify high quality firms with probability γ when following the leaders, a fraction γ^w of the wealthy who consume in any given period will purchase from a high quality firm. We will first show how γ^w is determined. If the wealthy themselves are the leaders, we then have to show that $\gamma = \gamma^w$, i.e. that a fixed point exists. Given also that the poor follow the leaders, a fraction γ^p of the poor who purchase in any given period will do so from a high quality firm. If the poor are the leaders, the fixed point will be $\gamma = \gamma^p$. The goal of this section is to establish conditions under which $\gamma^w > \gamma^p$ and vice versa. This will determine which group is being followed, since an uninformed consumer will follow the group with the higher γ^θ .

To calculate γ^θ , $\theta \in \{p, w\}$, we will consider each group individually. We will focus on the wealthy, but the derivations for the poor are analogous when replacing w with p . In the first period in which a wealthy person consumes, he samples a firm randomly and has probability λ of drawing a high quality. In every period thereafter he draws a high quality firm with probability γ whenever he searches. To derive the stationary distribution we must keep track of the proportion of wealthy consumers whose best quality encountered so far is q_l , q_h or \emptyset respectively, where \emptyset stands for those who have not yet consumed. In period t denote these by $n_l^{w,t}$, $n_h^{w,t}$ and $n_\emptyset^{w,t}$.

The relevant flow equations can then be constructed as follows: In period $t + 1$ consumers who have not tried any product include all newborns and those consumers who had not yet consumed at the beginning of period t , did not consume a product in period t and survived:

$$n_\emptyset^{w,t+1} = (1 - \delta) + \delta F(\hat{\rho}_\emptyset^w) n_\emptyset^{w,t}, \quad (3.2)$$

where $F(\hat{\rho}_\emptyset^w) > 0$ is the probability that a wealthy person who does not yet observe other market participants does not consume in the market. The cutoff $\hat{\rho}_\emptyset^w$ is analytically complicated,¹³ but our specification that in addition to the newborn, all consumers prior to their first purchase lack information about other market participants, eliminates $F(\hat{\rho}_\emptyset^w)$ in the derivation of γ^w .

Wealthy consumers in $t + 1$ who have state variable q_l include those without information at the beginning of period t who consumed a product in t , drew quality q_l and survived; those who had not found a satisfactory quality before t , drew quality q_l in t and survived; and those survivors from the prior period who did not consume a product

¹³For a given γ , the taste shock $\hat{\rho}_\emptyset^w$ is characterized by the indifference of the customer between going into the market and sampling a random firm vs. taking his outside option. If he goes into the market, his continuation payoff $EV_{\rho'}(q, \rho')$ is given in the appendix in (5.2) and (5.4). Let $X = \lambda[(1 - \delta)q_h + \delta EV_{\rho'}(q_h, \rho')] + (1 - \lambda)[(1 - \delta)q_l + \delta EV_{\rho'}(q_l, \rho')]$, then $\hat{\rho}_\emptyset^w \in (\underline{\rho}, \bar{\rho})$ is characterized by $[1 - \delta F(\hat{\rho}_\emptyset^w)][(1 - \delta)\hat{\rho}_\emptyset^w + X] = (1 - \delta)u_0^w + \delta \left[(1 - F(\hat{\rho}_\emptyset^w))X + \int_{\hat{\rho}_\emptyset^w}^{\bar{\rho}} \rho dF(\rho) \right]$.

and had experienced quality q_l before:

$$\begin{aligned} n_l^{w,t+1} = & \delta [1 - F(\hat{\rho}_\emptyset^w)] (1 - \lambda) n_\emptyset^{w,t} \\ & + \delta [1 - F(\hat{\rho}_l^w)] (1 - \gamma) n_l^{w,t} \\ & + \delta F(\hat{\rho}_l^w) n_l^{w,t} \end{aligned} \quad (3.3)$$

The cutoff $\hat{\rho}_l^w$ is given by (3.1) under the opportunity cost u_0^w .

Finally, the people who have state variable q_h constitute

$$\begin{aligned} n_h^{w,t+1} = & \delta [1 - F(\hat{\rho}_\emptyset^w)] \lambda n_\emptyset^{w,t} \\ & + \delta [1 - F(\hat{\rho}_l^w)] \gamma n_l^{w,t} \\ & + \delta n_h^{w,t} \end{aligned} \quad (3.4)$$

which is similar to (3.3) except for the last line: Since at q_h people return to the same firm, their state variable is q_h regardless of whether they consumed last period.

Stationarity is characterized by $n_\omega^{w,t'} = n_\omega^{w,t} = n_\omega^w$ for all t and t' and $\omega \in \{\emptyset, l, h\}$. We can use equations (3.2) to (3.4) to get

$$n_\emptyset^w = \frac{(1 - \delta)}{1 - \delta + \delta A^w}, \quad (3.5)$$

$$n_l^w = \frac{\delta n_\emptyset A^w (1 - \lambda)}{1 - \delta + \delta \gamma B^w}, \quad (3.6)$$

and

$$n_h^w = \frac{1}{1 - \delta} \frac{\delta n_\emptyset A^w [(1 - \delta) \lambda + \delta \gamma B^w]}{1 - \delta + \delta \gamma B^w}, \quad (3.7)$$

where $A^w \equiv [1 - F(\hat{\rho}_\emptyset^w)]$ represents the frequency of consumption for a wealthy consumer who has never consumed the product before, and $B^w \equiv 1 - F(\hat{\rho}_l^w)$ represents the frequency of consumption for a wealthy consumer who has only experienced low quality firms. Since γ^w represents the fraction of wealthy consumer who consumed in a period who have found a high quality firm, we must find the measure of consumers who actually consume in any given period. Denote by φ_l^w (φ_h^w) the measure of wealthy consumers who consume at low quality (high quality) firms in any given period in the steady state.

In any period the consumers who consume at q_l are all the uninformed players n_\emptyset^w who consume with probability $A^w = [1 - F(\hat{\rho}_\emptyset^w)]$ and draw a low quality with probability $(1 - \lambda)$, plus all informed players who have not found a high quality firm, n_l^w , who consume with frequency $B^w = [1 - F(\hat{\rho}_l^w)]$ and draw a low quality firm with probability $(1 - \gamma)$. Thus we have

$$\varphi_l^w = n_\emptyset^w A^w (1 - \lambda) + n_l^w B^w (1 - \gamma). \quad (3.8)$$

For φ_h^w , we have similar terms for the uninformed and unsatisfied players, plus an additional term reflecting those people who had already found a firm of satisfactory quality in the past, n_h^w , times their frequency of consumption $C^w \equiv [1 - F(u_0^w - q_h - s_h(w))]$. Therefore

$$\varphi_h^w = n_\emptyset^w A^w \lambda + n_l^w B^w \gamma + n_h^w C^w.$$

Since $\gamma^w = \frac{\varphi_h^w}{\varphi_l^w + \varphi_h^w}$ we get

$$1 - \gamma^w = \frac{\varphi_l^w}{\varphi_l^w + \varphi_h^w} = \frac{n_\emptyset^w A^w (1 - \lambda) + n_l^w B^w (1 - \gamma)}{n_\emptyset^w A^w + n_l^w B^w + n_h^w C^w}, \quad (3.9)$$

where again A^w , B^w and C^w represent the frequencies of consumption for those wealthy consumers who have not consumed from any firm before, those who have only experienced low quality firms and those who have previously experienced a high quality firm, respectively. Substituting the value for n_\emptyset^w , n_l^w and n_h^w from equations (3.5) to (3.7) and rearranging yields an expression for γ^w that is independent of the initial frequency A^w , i.e.

$$\gamma^w = 1 - \frac{[1 - \delta + \delta B^w] [1 - \lambda]}{1 - \delta + \delta(1 - \lambda + \gamma) B^w + \left[\delta \lambda + \frac{\delta^2}{1 - \delta} B^w \gamma \right] C^w} \quad (3.10)$$

for a given $\hat{\rho}_l^w$, which is implicitly defined in equation (3.1) in lemma 2.

The proof of the following lemma, which is left to the appendix, shows that there is a fixed point $\gamma^w = \gamma$.

Lemma 4 *There exists a fixed point $\gamma^w = \gamma$, $\gamma \in (\lambda, 1)$, in equation (3.10) such that equation (3.1) is also satisfied.*

We argued above that it is always better to follow some group than to sample randomly. Therefore in a stationary equilibrium, $\gamma = \gamma^w$ or $\gamma = \gamma^p$, where γ^p is constructed analogously. While an uninformed consumer does better by following any other consumer than searching randomly, one should expect that it is better to follow a wealthy consumer than a poor consumer. Suppose that the level of service offered to wealthy consumers is at least as large as the service offered to poor consumers. Then while uninformed, the wealthy will consume more frequently than the poor, and hence, will discover a high quality firm more quickly.

While this argument is appealing, it is not trivial. The wealthy search more often at low quality firms in any period, diluting the visibility of the wealthy who have identified a high quality firm. Partial differentiation of (3.10) reveals that $\partial \gamma^w / \partial B^w > 0$ if $\gamma > \lambda$, and therefore the dilution effect does not outweigh the greater frequency with which they consume. If the frequency of consumption while searching for a high quality firm increases for either group, this unambiguously increases the quality of their signal.

Nevertheless, the visibility induced through the frequency of consumption is important: If the poor is the group that receives service (which we will show is only possible if service is sufficiently valuable), then it is possible that the poor consume less frequently prior to identifying a high quality firm than the wealthy, but it is nevertheless better to follow the poor. The reason is that the poor consume much more frequently at high quality firms, and therefore those who have identified a high quality firm are a greater proportion than the proportion of wealthy who have identified a high quality firm.

We summarize this in the following proposition the proof of which is left to the appendix. Recall that ξ_γ was introduced in lemma 3 as the threshold on service below which the poor search less frequently at low qualities even if they are the ones that receive service.

Proposition 1 *Let $\delta \in (\underline{\delta}, 1)$ and $\gamma \in (\lambda, 1)$. Consider a candidate stationary equilibrium with $s_h(\cdot) \geq s_l(\cdot)$. Then there exists $\hat{\xi}_\gamma \in (u_0^p - u_0^w, \xi_\gamma)$ such that*

- 1) *If $s_h(w) \geq s_h(p)$, then $\gamma^w > \gamma^p > \lambda$.*
- 2 a) *If $s_h(p) - s_h(w) = \bar{s} < \hat{\xi}_\gamma$, still $\gamma^w > \gamma^p > \lambda$.*
- 2 b) *If $s_h(p) - s_h(w) = \bar{s} > \hat{\xi}_\gamma$, then $\gamma^p > \gamma^w > \lambda$.*

The proposition establishes a bound on the importance of service below which all consumers will follow wealthy consumers. Only if \bar{s} exceeds this bound could it be preferable to follow the poor.

3.3. Firms' behavior

We next analyze service provision by the firms. We establish that it is profitable for the firm to provide service to some type of consumers if they are followed by sufficiently many consumers of the other type. We also prove that high quality firms will always provide at least as high a service level as low quality firms, which we have so far taken as exogenous. A key observation for this result is that in any stationary equilibrium, if a high-quality firm promises in any period service \bar{s} to a consumer, then the consumer will return to this firm the next period he enters the market, regardless of his expectations about future service. The intuition behind this observation is that accepting the optimal per-period outcome of high quality and high service and searching thereafter for a new firm yields higher utility than searching immediately. Therefore, high-quality firms can always ensure the return of a consumer by promising him service. The question in this section is when this will be profitable.

Our first result concerns the equilibrium profit contribution of a customer. In a stationary equilibrium, a consumer who purchases from a firm has the same expected

number of followers in every period. Thus, the benefit to a firm of a single visit of a particular customer is the following: He pays price P , potentially receives service at cost $c > P$, and induces the expected discounted lifetime equilibrium profit that the firm generates from his next-period followers. Call this benefit Π . Π is an equilibrium object that depends on the strategies of the firm in question as well as the strategies of other firms and consumers. If a firm deviates and promises s' instead of the equilibrium promise s , the benefit of the next return visit is $\Pi - (c(s') - c(s))$. Since in a stationary equilibrium after a one shot deviation the continuation game is unchanged once the customer returns, only the immediate cost of service changes from $c(s)$ to $c(s')$. In particular, the behavior of the customer once he returns as well as the behavior of the followers is unchanged.¹⁴ For example, promising no service instead of service in any period changes the benefit of the next visit of the consumer from Π to $\Pi + c$, as the firm saves the service costs next time. Yet it might delay the consumer's return, as now consumption is less valuable compared to the opportunity cost of consumption. If the consumer switches to a competitor, $\Pi + c$ will in fact never be realized.

Consequently, if a particular consumer generates sufficient indirect profit it will pay a firm to promise that consumer service to induce him to purchase more frequently. In addition, Proposition 1 provides conditions under which it is optimal to follow consumers of type θ . If the fraction of this leading group is not too large, the spillover benefits of the followers will make it profitable for high quality firms to provide service to the leaders. We state this formally in the next proposition.

Moreover, we establish that it is indeed optimal for high quality firms to outbid low quality firms in their pursuit of valuable customers. We show that if it is profitable for a low wage firm to provide service to a consumer, it is also profitable for a high wage firm to provide service in order to keep the business of this consumer. In Lemma 1 we showed that if consumers are sufficiently long-lived, that is, $\delta \in (\underline{\delta}, 1)$, they will continue to search for a high quality firm rather than return to a low quality firm. This was derived under the assumption that high quality firms offer at least weakly more service. The result here establishes that this assumption is indeed fulfilled. We summarize this in the following proposition, the proof of which is relegated to the appendix. For more compact notation of the proposition, let $\alpha^w = \alpha$ and $\alpha^p = 1 - \alpha$.

Proposition 2 *Let $\delta \in (\underline{\delta}, 1)$.*

¹⁴This is a consequence of the assumption of markov perfection which is embedded in the requirement that the equilibrium service strategy does not depend on past histories. In the situations that we analyze the consumer is not indifferent between his choices of searching for a new firm or returning to a previous firm. Furthermore, the decision situation is effectively unchanged compared to on the equilibrium path play once he returns after a one-shot deviation. Taken together, this implies that his continuation strategy once he returns will be his on-the-equilibrium-path strategy.

- i) Suppose all uninformed consumers follow consumers of type θ . Then there exists $\bar{\alpha} > 0$ such that for $\alpha^\theta < \bar{\alpha}$, in any stationary equilibrium $s_h(\theta) = \bar{s}$.
- ii) In any stationary equilibrium, either $s_h(\theta) \geq s_l(\theta)$ for $\theta \in \{p, w\}$; or $s_h(\theta) < s_l(\theta)$ but type θ consumers nevertheless do not return to low quality firms.

We should point out that two forces can lead to high-quality firms' willingness to provide service. One is competitive pressure. If other high-quality firms offer service, then if a single firm does not provide service, the consumer might not return, preferring to search for another firm that provides service. The second is the encouragement effect, that is, the consumer returns more frequently when he is offered service. In our model the encouragement effect is important to sustain a high-service equilibrium. That is due to the Markovian assumption on the firms' strategies which implies that service provision only depends on the consumer type, and not on the firm's own or other players' past actions. In an equilibrium in which the high-quality firms are supposed to promise service, the firm's strategy specifies offering service again in the continuation game even if it deviated for a period and did not offer service. After a deviation a consumer will therefore still expect to get a service promise the next time he consumes there (even though he will not get any actual service provided in that period) and it is better for him to return than to search for a new firm in which he also will not receive any service in the first period of consumption, but might draw a low quality. Therefore, the consumer would return even if service is not provided for one period. The motivation for high quality firms to offer service then comes from the encouragement effect. If we impose less stringent assumptions on the equilibrium (in particular, dropping the restriction to Markov strategies), the competitive forces can be very important. If consumers believe that they will not be promised service by a firm ever again if it does not promise service in a given period, then service provision can be sustained by the competitive threat that consumers search for new firms that do offer service, which seems intuitively appealing.

With these results on optimal strategies for consumers and firms and the steady state derivations we now turn to the stationary equilibria of the game.

3.4. Equilibria

We first provide a necessary condition for equilibria when the value of service is not too large. In any such equilibrium, the poor follow the wealthy, and if service is provided, it is provided only to the wealthy. This is driven by the fact that the wealthy accumulate information faster than the poor.

Proposition 3 *Let $\delta \in (\underline{\delta}, 1)$. There exists $\xi > u_0^p - u_0^w$ such that for $\bar{s} < \xi$ in any stationary equilibrium all uninformed consumers follow wealthy consumers after their initial purchase. If service is provided in equilibrium, it is only provided to the wealthy.*

Proof: By proposition 2 higher quality firms provide weakly higher service. In combination with lemma 1 we know that all consumers search for high quality firms. That is, all consumers, wealthy and poor, will in equilibrium follow the distribution that places the highest weight on high quality firms. By Proposition 1 $\bar{s} < \hat{\xi}_\gamma$ ensures that all consumers will follow the wealthy, even if the poor receive the service. Since $P < c$ service is only provided (i.e., promised and then delivered) to players who have followers. Therefore, in equilibrium only the wealthy can receive service. While $\hat{\xi}_\gamma$ depends on γ , one can show that it is bounded away from $u_0^p - u_0^w$ for all $\gamma \geq \lambda$. *Q.E.D.*

Whether stationary equilibria with or without service exist depends on the profit generated by the followers that a consumer has. Since $P < c$ service will only be provided when the profit generated by one's followers is sufficiently large. We establish existence and uniqueness separately for the cases when there are many or few poor.¹⁵ With many poor people, the wealthy arise unambiguously as the leaders and will be provided with service to further encourage their search externality. With few poor people there still is always an equilibrium in which everybody follows the wealthy, but no service is provided. Only if service sufficiently outweighs the difference in the opportunity costs can the poor receive service and be the leaders.

When analyzing the equilibria, we will discuss cases where a consumer is followed mainly by consumers of his own type. It is therefore important to understand whether this induces service or not. Consider the case where there is only one group of consumers. Consumers without followers do not receive service, since the price of the good is lower than the cost of providing service. If consumers do not die, i.e., $\delta = 1$, there cannot be a steady state in which a non-trivial fraction of consumers has not found a high quality firm. Therefore in any steady state equilibrium, consumers do not search for new firms, and thus consumers do not have followers. Therefore no firms will find it profitable to provide service to any consumer, independently of the service provided by other firms. By continuity, there exists $\delta^* < 1$ such that this holds for all $\delta \in (\delta^*, 1)$, where δ^* is a function of the price P and the cost c .¹⁶ That is, there exists a value δ^* such that for survival rates greater than δ^* , no firm would find it profitable to provide service to consumers who are only followed by consumers of their own type, independent of the service promises of the other firms.

¹⁵We have not shown that the fixed point distribution when the rich follow themselves is unique. Also, service by low quality firms might make no difference to the consumers' search. Therefore equilibria might not be unique, but all exhibit the properties we want to establish.

¹⁶This can also be seen by considering equation (5.9) with $N_p = 0$ and $N_r = 1 - \gamma$, where γ is the fixed point to equation (3.10). Since $1 - \gamma$ converges to 1 and $\frac{1-\gamma}{1-\delta}$ to $\frac{1}{1-F(u_0^r - q_h - \bar{s})}$ for δ converging to 1, $\Pi^{rh} \approx (P - c) < 0$ for δ large. The argument can be extended to low quality firms and poor consumers, and to the case where other firms do not provide service. It remains valid even when $\beta = \delta$, i.e., the firms' discount factor is also large.

The following proposition establishes the existence of equilibria in an environment when the ratio of poor to wealthy consumers is sufficiently large. All equilibria exhibit service only for the wealthy customers if the survival rate is sufficiently high or service is lower than the threshold ξ in Proposition 3.

Proposition 4 *Fix $\delta \in (\underline{\delta}, 1)$ and $\bar{s} > 0$. There exists $\alpha^* \in (0, 1)$ such that for $\alpha \leq \alpha^*$ the following holds:*

- 1) *There exist stationary equilibria in which all consumers follow wealthy consumers while searching. Consumers stop searching only when they find a high quality firm, and high quality firms offer service to the wealthy and not to the poor. Low quality firms may offer service but do not attract repeat business.*
- 2) *All stationary equilibria are of this form if $\bar{s} < \xi$ or if $\delta > \delta^*$ and α^* sufficiently small.*

Proof: Assume all players follow the wealthy when searching. Then the poor will never be promised service by any firm that expects repeat business, as by $P < c$ the firm would make a loss. The wealthy will be promised service by all high quality firms. These firms can induce the consumer to return by offering service. For α^* small enough Proposition 2 establishes that this will be the only choice that does not have a profitable deviation. By lemma 1 low quality firms are never repeatedly visited by a wealthy player. It is immediate that all players have an incentive to follow the wealthy: since $s_h(w) \geq s_h(p)$ by Proposition 1 $\gamma^w > \gamma^p > \lambda$, and following the wealthy is better than following the poor or sampling randomly.

For $\bar{s} < \xi$ no other equilibria exist, as by Proposition 3 all players follow the wealthy and the assumption of the prior paragraph is fulfilled. If $\delta > \delta^*$, then it is not profitable to provide service to the poor if they are followed only by other poor consumers. If α^* is sufficiently small, then each poor consumer can only have a negligible number of wealthy followers, and providing service to the poor remains unprofitable even if all consumers follow the poor. If the poor do not receive service, they prefer to follow the wealthy, and again the assumption of the prior paragraph is fulfilled. *Q.E.D.*

The proposition shows that firms indeed support the learning process when the service can be concentrated on sufficiently few wealthy people who achieve a high visibility in the market. For all consumers the outcome is clearly preferred to a world in which service is absent. Wealthy consumers benefit directly from the service and indirectly because they obtain high qualities faster. Poor customers benefit also, but only indirectly through the improved search externality provided by the wealthy. High quality firms benefit, because consumers find high quality firms faster. Yet their cost of providing service might outweigh this benefit. Low quality firms unambiguously lose compared to a

world without firms' ability to interference with the consumers search process. Service increases the informational externality between consumers, and a newborn consumer tries on average fewer low quality firms before finding high quality.

As a comparison we analyze the case in which the ratio of wealthy to poor players is reversed. If there are few poor people, there is still always an equilibrium in which everybody follows the wealthy. Only if service is sufficiently important is there also a second equilibrium in which everybody follows the poor.

Proposition 5 *Fix $\delta > \max\{\underline{\delta}, \delta^*\}$ and $\bar{s} > 0$. There exists $\alpha^{**} \in (0, 1)$ such that for $\alpha \geq \alpha^{**}$ the following holds:*

- 1) *There exist stationary equilibria in which all consumers follow wealthy consumers while searching. Consumers stop searching only when they find a high quality firm. High quality firms do not offer service to any consumer. Low quality firms may offer service but do not attract repeat business.*
- 2) *If $\bar{s} < \xi$ these are the only equilibria. There is $\xi' > u_0^p - u_0^w$ such that there also exist equilibria in which all consumers follow poor consumers while searching if $\bar{s} > \xi'$. Consumers stop searching only when they find a high quality firm. High quality firms offer service to the poor and not to the wealthy. Low quality firms may offer service but do not attract repeat business.*
- 3) *There do not exist stationary equilibria with other properties.*

Proof: Assume all consumers follow the wealthy. Since $\delta > \delta^*$ the wealthy do not receive service due to wealthy followers, and α^{**} small enough assures there will not be sufficient poor followers to warrant service.¹⁷ Also the poor do not get service. Proposition 1 then establishes $\gamma^w > \gamma^p > \lambda$, and indeed everybody wants to follow the poor. By Proposition 3, for $\bar{s} < \xi$ there cannot be any other stationary equilibria in which the wealthy are not being followed.

Consider a stationary equilibrium in which the poor do not follow the wealthy. It must then be the case that the wealthy follow the poor. If the wealthy did not follow the poor, the poor would not receive service, and everybody would follow the wealthy as in the previous paragraph. If the wealthy follow the poor, then by lemma 2 high quality firms would indeed want to provide service to the poor. Yet the wealthy will only follow the poor if $\gamma^p \geq \gamma^w$. By Proposition 3 this only happens for $\bar{s} \geq \hat{\xi}_{\gamma^p}$. Since γ^p is bounded away from one for all \bar{s} since some newborns are always searching, it

¹⁷The contribution by the poor is $\delta \frac{\varphi_l^p}{\varphi_l^w + \varphi_h^w} \frac{1-\alpha}{\alpha} P \left[1 + \frac{\delta\beta}{1-\delta\beta} (1 - F(u_0^p - q^h)) \right]$. Since for a given δ the term $\frac{\varphi_l^p}{\varphi_l^w + \varphi_h^w}$ is bounded from above and independent of α , the expression converges to zero as α converges to one.

is easy to see that $\hat{\xi}_{\gamma^p}$ is bounded. Therefore there exists ξ' such that $\bar{s} \geq \xi'$ implies $\gamma^p \geq \gamma^w$. *Q.E.D.*

Proposition 5 reveals the natural advantage that the wealthy possess in information gathering. Following the wealthy is always an equilibrium, as in the absence of service it is best for everybody to follow them. Only if service is very attractive will the poor search enough such that following them can be worthwhile for the wealthy. As discussed in the context of Proposition 1, service has to sufficiently outweigh the difference in opportunity costs, but does not need to be so high that poor consumers actually find high qualities faster than wealthy consumers. It is worth noticing that in the case where both types of equilibria coexist, consumers are all better off in the equilibrium where consumers follow the poor and the poor obtain service.

Propositions 4 and 5 establish that it is the information that is revealed in the choices of the wealthier players that makes them valuable to other players and, by extension, to firms. If there are sufficiently many consumers who value this information, the wealthy are in a unique position to profit from this if service is not too valuable. Poor consumers are not substitutes for the wealthy as their actions reveal less information than those of the wealthy, even if the visibility of the poor is much better when there are fewer of them. Note that we have effectively ruled out trigger strategies in the analysis.¹⁸ Hence, firms' decisions are primarily influenced by the per period contribution of a customer. Thus, it is not the frequency of consumption per se that allows wealthier consumers to command service, but rather the induced information that is valued by other consumers, and in turn by the firms.

4. Discussion

The mechanics of our model are sufficiently transparent to allow the discussion of additional social components such as conspicuous consumption, visibility and the importance of relative position in society. We discuss these in the next subsection. We discuss the robustness of our results to various changes in the model assumptions in the following subsection and then conclude.

4.1. Social Interaction

Our interpretation of the different opportunity costs u_0^w and u_0^p has been derived from differences in income that affect the consumers' budget constraints. For the analysis, u_0^w and u_0^p could be taken as primitives that result from heterogeneity with respect to

¹⁸These would have allowed richer customers to impose harsher punishment on firms, as their overall lifetime consumption is higher and their effective discount factor is higher due to more frequent consumption.

something other than income. They could, for example, reflect differences in tastes. If you look for a Swedish restaurant, Swedes might have a greater preference than the average consumer, that is, have lower u_0 . For running shoes, runners will consume more, and good jazz places are likely most likely discovered by following jazz enthusiasts. While our analysis can easily handle exogenous differences, our focus on income differences stems from two observations. For normal goods income differences will induce higher consumption for the wealthy. More importantly, in many situations income differences might be easier to infer than differences in taste. If taste heterogeneity is similar for different income categories but only income differences are observable, then the firms' treatment decisions and the consumers' decisions on whom to follow will be based on the observable characteristic.

The ability to distinguish between wealthy and poor is important in our framework. Typically this must be inferred from some attribute, for example from the suit one wears or the car one drives, suggesting a rational basis for conspicuous consumption.¹⁹ A standard signalling argument would explain why those who would like to consume more frequently would rationally choose to spend the money for a Rolex watch if it leads to greater service while less frequent purchasers would not. It is interesting to note that the inefficiencies associated with the excess spending on such items is at least partially offset by the increased efficiency in the search process made possible by the conspicuous consumption.

Our results also highlight the importance of visibility in the marketplace. Given our signal technology, a consumer of the group that is relatively small is most visible. Therefore it is the small group that can receive service, as service is tied to a sufficient number of followers. This can obviously be extended to a setting in which consumers in the same income category have different visibility in the market.²⁰ Again those with higher visibility are more likely to receive service.

As a final point it should be noted that our concept of wealthy vs. poor is one of relative comparison. Being materially better off than others is important, as this results in a "leader" status. The absolute level is not crucial for this. Thus, even in market settings relative comparisons can be important.²¹ This provides an understanding of how some groups can enjoy leadership status even when there is only slight heterogeneity in society. The actual market outcome in terms of consumption might be quite different even though the underlying heterogeneity is small, because firms may interact with the

¹⁹Fang (2001) analyses conspicuous consumption as a way to mediate informational free-riding in a labor market context; Cole, Mailath and Postlewaite (1995) investigate this in a matching setting.

²⁰Assume e.g. two subgroups of wealthy players, and each consumer sees a member of the first subgroup for sure but a member of the second only with probability smaller than one.

²¹Cole, Mailath and Postlewaite (1992) and Mailath and Postlewaite (2002) provide a rationale for relative comparisons in a model where benefits of higher relative standing arise from more attractive matching opportunities. Samuelson (2004) attributes relative comparisons to evolutionary pressure.

search process in ways that magnify intrinsic heterogeneity.

4.2. Robustness

We will discuss some of the modelling choices we have made and the robustness of our results to changes in these assumptions. One of the features of our model that deserves discussion is the validity of the commitment. We assumed that firms promise some service level and always honor their promise when the customer returns. This commitment assumption is a shorthand way of introducing reputational considerations that allow for service provision even in Markov-perfect equilibria, i.e., it allows us to rule out equilibria in which service is not provided because firms cannot convince consumers that they will get service in the future. In the analysis we have not considered whether firms would want to renege on their promise (as we assumed that this is not possible). Commitment is not necessary to support this equilibrium in a world without commitment if we allow non-Markovian (trigger) strategies: incentive compatibility of the commitments can be ensured. Equilibria without service as in Proposition 5 can trivially be supported by out of equilibrium beliefs that no firm will provide service in the future, even if it provided service in the current period. Providing service in any period thus only induces costs to the firm without altering future benefits. Equilibria with service such in Propositions 4 and 5 can be supported if consumers believe to never receive service again from a firm that deviates from equilibrium service provision. If players are sufficiently long-lived and patient, the loss of the consumers business or the slowdown of his visits still warrant service provision. Obviously, uniqueness claims do not apply to such a non-restricted environment.

In the equilibrium analysis we have established existence for small and large fractions α of wealthy consumers. Due to our restriction to pure markovian strategies we cannot ensure existence for intermediate α for arbitrary parameters c , \bar{s} and $F(\cdot)$. If service is provided, the signal is better and consumers search more efficiently, which reduces the number of followers. Therefore with service the number of followers might be too small to warrant service, while without service the number of followers might be large and service is profitable. In these cases mixed or non-markovian strategies would be necessary for existence.

We also limit attention to two quality levels. This assumption is not entirely innocuous. If there were three qualities $q_l < q_m < q_h$ with associated population fractions λ_l , λ_m and λ_h , it is possible that wealthy consumers search only for q_h firms, while poor consumers stop if they found a medium quality firm since their lower frequency of consumption acts similarly to a lower discount factor. Following the wealthy then implies a high probability of drawing a high quality firm, but also a relatively high probability of finding a low quality firm since the wealthy do not settle on a medium quality firm should they find one, and hence may search for a long time. Following the poor reduces

the risk of finding a low quality firm if they stop searching once they identify either a medium or high quality firm. This can lead to the wealthy following the wealthy to obtain high quality, and the poor following the poor to find medium or high quality firms. Nevertheless, modified versions of our results hold if the survival rate δ is sufficiently large, since in that case both poor and wealthy will continue searching until they find a high quality firm.

We also restricted attention to only two levels of service. This simplifies the analysis, but the model could accommodate multiple levels of service s_1, \dots, s_n at costs c_1, \dots, c_n . In Proposition 4 the level of service to the poor would be small even if they are followed, because the number of followers is small. Therefore they would nevertheless search less frequently, only the wealthy are followed and substantial service is only given to the wealthy. In Proposition 5 the equilibrium in which the wealthy are followed continues to exist, because without followers the poor receive little service and the wealthy consume more often. In this case all consumers receive little service. There will also be an equilibrium in which everybody follows the poor, as they then have many followers. Many followers will induce firms to provide top service, and high quality firms outbid low quality firms. This also happens if service is a continuous choice $s \in [0, \bar{s}]$.²² $c_n > P$ or $c(\bar{s}) > P$ would again insure that maximum service is not simply given due to individual consumption. Yet individual consumption could warrant a service level above zero.

We have assumed a simple signal structure in which each consumer can observe one wealthy and one poor consumer in every period. All results hold if a consumer can observe multiple wealthy and multiple poor consumers each period. Since there is a continuum of firms, the probability of observing two or more consumers who choose the same firm is zero. Therefore several consumers of the same type are as informative as a single consumer, and an individual simply decides to follow either one of the wealthy or one of the poor. On the other hand we could have assumed that each consumer simply sees the choices of N randomly chosen consumers who purchased last period. This assumption is closer to models such as those analyzed by Ellison and Fudenberg (1995) and Banerjee and Fudenberg (2003). While analytically much more complicated, we do not think that it changes the flavor of the results as long as N is sufficiently large given the fraction of wealthy people. The reason is that a consumer only cares about observing at least one poor or at least one wealthy person, depending on whom he wants to follow. For sufficiently large N , the probability is virtually one that one poor and one wealthy consumer is in the observed set.

²²If the service is unbounded, there are parameter constellations for which low quality firms provide service so high as to prevent the leaders from continued search for high qualities and a pure strategy equilibrium may not exist. The reason is that low quality firms are willing to give up the full surplus to retain the customer, while high quality firms are only driven by the encouragement effect of more frequent consumption due to the markovian restriction.

Some of the social learning literature assumes some private information by agents. If we assume that agents receive in each period a private signal that indicates a firm which is with probability ψ of good and with $1 - \psi$ of average quality, this would change the probability of finding a high quality firm when sampling independently to $\lambda' = \psi + (1 - \psi)\lambda$, rather than simply λ , leaving the results qualitatively unchanged.

We took the price as being exogenously set, and identical across firms regardless of quality. We argued above that it seems unrealistic that even if prices differed across firms, they would perfectly convey the quality of firms, and there would remain the possibility that social learning of the sort in our model would still play a role. It is nevertheless worth discussing what the equilibria of a model such as we have laid out would look like if prices were a strategic variable rather than exogenously set. Suppose that there were a symmetric equilibrium in which all low quality firms set one price and all high quality firms set a possibly different price. If the difference in quality between the high and low quality firms is small, there may be a separating equilibrium in which the prices of the two types of firms are not very different, and wealthy people go to high quality firms while the poor go to cheaper, low quality firms. Suppose, however, that there was no value to the low quality firm; that is, even if the price were zero, all consumers would prefer the high quality firm. There clearly cannot be a separating equilibrium then since low quality firms could profitably charge the same price as high quality firms. If all firms charge the same price, whether any single firm has an incentive to deviate depends on consumers' beliefs when they see an out-of-equilibrium price. Trivially, beliefs that it is a low quality firm that deviates will support equal pricing.²³ Hence, if our model were extended so that pricing was endogenized, one would get the equal pricing that we assumed.

5. Appendix

It will be convenient to prove lemma 2 prior to lemma 1.

Proof of Lemma 2: We consider first the case where the consumer is promised the same service $s \in \{0, \bar{s}\}$ from every firm in every period. Having characterized the consumer search behavior for this case, it is straightforward to extend it to the case where low-quality firms promise less. We will work with average discounted payoffs. The functional equation for sampling with recall, given that the best quality the consumer

²³For low quality firms only new customers are important. These firms want to pool on any price above marginal cost if they otherwise get no new customers. High quality firms obtain profits from new customers, but also from existing customers. Analogous to the Diamond Paradox, existing customers face switching costs and a high quality firm can extract rents from them. In equilibrium high quality firms must charge a price such that a deviation does not increase the profits from existing customers beyond the loss of sales due to absence of new customers.

has yet encountered is q , and given the current shock ρ , can be written as

$$\begin{aligned} V^C(q, \rho) = & \max\{(1 - \delta)(q + s + \rho) + \delta E_{\rho'} V^C(q, \rho'), \\ & (1 - \delta)(E_{q|\gamma}(q) + \rho) + \delta E_{\tilde{q}|\gamma} E_{\rho'} \max\{V^C(q, \rho'), V^C(\tilde{q}, \rho')\}, \\ & (1 - \delta)u_0 + \delta E_{\rho'} V^C(q, \rho')\}, \end{aligned} \quad (5.1)$$

where the first line describes the utility from returning to a known firm with quality q , the second line describes random sampling and the last line consumption of the numeraire. E_x denotes the expectation operator with regard to variable x . $x = q|\gamma$ refers to variable q when the probability of a high quality is γ .²⁴ We drop the decision-irrelevant constant $u(y - P)$. The right hand side of (5.1) defines an operator $T : Z \rightarrow Z$. If $[\underline{\rho}, \bar{\rho}]$ is bounded, $Z = \{\nu : \{q_l, q_h\} \times [\underline{\rho}, \bar{\rho}] \rightarrow \mathbb{R} | \nu \text{ is continuous and bounded}\}$ and it is easily checked that T fulfills Blackwell's (1965) sufficient conditions for a contraction. Therefore, a solution to the problem exists and is unique. For $\bar{\rho} = \infty$ note that for all $\rho > \bar{\rho}_r = u_0 - q_l + \frac{\delta}{1-\delta}(q_h + \bar{s} - q_l)$ the consumer will consume the indivisible good, because even if he loses high quality and high service forever (and only gets q_l now instead of u_0) the taste shock today outweighs the forgone benefits. Therefore we do not alter his decision problem if we restrict $[\underline{\rho}, \bar{\rho}]$ to $[\underline{\rho}, \bar{\rho}_r]$ and assume a distribution $F_r(\rho) = F(\rho)$ for all $\rho < \bar{\rho}_r$ and $F_r(\rho) = 1$ for all $\rho \geq \bar{\rho}_r$. Similarly, for $\rho < \underline{\rho}_r = u_0 - q_h - \bar{s} - \frac{\delta}{1-\delta}(q_h + \bar{s} - q_l)$ the consumer would always choose the numeraire to avoid the negative taste shock, even if he were certain to permanently gain high quality and high service by consuming the indivisible good, and we can bound the shock distribution from below without altering the consumer's decision. On the restricted problem the contraction property establishes that (5.1) has a unique solution, and so the unrestricted problem has a unique solution.

From (5.1) note that $V^C(q, \rho)$ is weakly increasing in q . Therefore for $q = q_h$ the first line in the max-operator is larger than the second. Thus, whenever a consumer with state variable q_h enters the market, he will return to the firm with quality q_h rather than sample a new one. He enters the market if the taste shock is high enough, i.e., higher than $\hat{\rho}_h \in (\underline{\rho}, \bar{\rho})$ that makes the player indifferent between not consuming (line 3 in equation (5.1)) or going into the market (line 1), so that

$$(1 - \delta)(q_h + s + \hat{\rho}_h) + \delta E_{\rho'} V^C(q_h, \rho') = (1 - \delta)u_0 + \delta E_{\rho'} V^C(q_h, \rho')$$

or $\hat{\rho}_h = u_0 - q_h - s$.

Then in any given period the ex ante probability that this player will enter the market is $[1 - F(u_0 - q_h - s)]$, while the ex ante probability of not consuming is $F(u_0 - q_h - s)$.

²⁴We used the shortcut γ for $q|\gamma$ in the main body of the text.

Knowing this, the expected average discounted payoff is

$$\begin{aligned}
E_{\rho'} V^C(q_h, \rho') &= \int_{\underline{\rho}}^{\bar{\rho}} \max\{q_h + s + \rho, u_0\} dF(\rho) \\
&= F(u_0 - q_h - s)u_0 + [1 - F(u_0 - q_h - s)](q_h + s) + \int_{u_0 - q_h - s}^{\bar{\rho}} \rho dF(\rho) \\
&= u_0 + \int_{u_0 - q_h - s}^{\bar{\rho}} [1 - F(\rho)] d\rho.
\end{aligned} \tag{5.2}$$

The second equality is simply the probability of not consuming times the opportunity cost, plus the probability of going into the market times the value from quality and service of doing so, plus the expected value of the taste shock when going into the market. The last line follows by integration by parts.

Now consider $q = q_l$. Assume that searching for a higher quality firm is preferable to returning to the low quality firm and obtaining service in the next period. (We will show in the subsequent proof of lemma 1 that this is indeed optimal). The threshold $\hat{\rho}_l$ for the taste shock is now given by the equality of line 2 and 3 in (5.1), so that

$$(1 - \delta)(E_{q|\gamma}(q) + \hat{\rho}_l) + \delta(1 - \gamma)E_{\rho'} V^C(q_l, \rho') + \delta\gamma E_{\rho'} V^C(q_h, \rho') = (1 - \delta)u_0 + \delta E_{\rho'} V^C(q_l, \rho'),$$

or

$$\frac{\delta\gamma}{1 - \delta} [E_{\rho'} V^C(q_h, \rho') - E_{\rho'} V^C(q_l, \rho')] = u_0 - E_{q|\gamma}(\tilde{q}) - \hat{\rho}_l. \tag{5.3}$$

Taking $\hat{\rho}_l$ as given, we can express the expected value as

$$\begin{aligned}
E_{\rho'} V^C(q_l, \rho') &= F(\hat{\rho}_l)[(1 - \delta)u_0 + \delta E_{\rho'} V^C(q_l, \rho')] \\
&\quad + [1 - F(\hat{\rho}_l)](1 - \delta) [E_{q|\gamma}(q) + E_{\rho'}(\rho' | \rho' \geq \hat{\rho}_l)] \\
&\quad + [1 - F(\hat{\rho}_l)]\delta [\gamma E_{\rho'} V^C(q_h, \rho') + (1 - \gamma)E_{\rho'} V^C(q_l, \rho')].
\end{aligned}$$

The first line weights the opportunity cost of consumption by the probability $F(\hat{\rho}_l)$ of not consuming. The term $[1 - F(\hat{\rho}_l)]$ in the second and third line reflects the probability of entering the market. The utility from doing so is comprised of two components. Line 2 reflects the instantaneous expected value from entering the market due to quality and taste shock, while line 3 represents the expected continuation value after encountering a firm with high or low quality respectively. After rearranging terms we have

$$\begin{aligned}
E_{\rho'} V^C(q_l, \rho') &= F(\hat{\rho}_l)u_0 + [1 - F(\hat{\rho}_l)]E_{q|\gamma}(q) + \int_{\hat{\rho}_l}^{\bar{\rho}} \rho dF(\rho) \\
&\quad + [1 - F(\hat{\rho}_l)]\frac{\delta\gamma}{1 - \delta} [E_{\rho'} V^C(q_h, \rho') - E_{\rho'} V^C(q_l, \rho')].
\end{aligned}$$

Inserting (5.3) and rearranging gives

$$E_{\rho'} V^C(q_l, \rho') = u_0 - [1 - F(\hat{\rho}_l)]\hat{\rho}_l + \int_{\hat{\rho}_l}^{\bar{\rho}} \rho dF(\rho) = u_0 + \int_{\hat{\rho}_l}^{\bar{\rho}} [1 - F(\rho)]d\rho. \quad (5.4)$$

Substituting (5.4) and (5.2) into (5.3), we obtain an implicit function characterizing the threshold shock value $\hat{\rho}_l \in (u_0 - q_h - s, u_0 - E_{q|\gamma}(q))$:

$$\hat{\rho}_l - u_0 + E_{q|\gamma}(q) + \frac{\delta\gamma}{1-\delta} \int_{u_0 - q_h - s}^{\hat{\rho}_l} [1 - F(\rho)]d\rho = 0. \quad (5.5)$$

By the intermediate value theorem there is a solution to this equation, and the solution is unique as the left hand side is strictly increasing in $\hat{\rho}_l$.

Finally, note that when both firms offer service $s = \bar{s}$, the customer will not return to a low quality firm (see lemma 1). Since service is not provided in the first period, the customer will never experience service from any low quality firm, even if it promises to provide service should the customer return. Therefore the results also hold for the case where only high quality firms promise service \bar{s} , while low quality firms may not. *Q.E.D.*

Proof of Lemma 1: Consider a consumer of type θ with opportunity cost u_0^θ who has experienced only low quality firms. If low quality firms do not offer service, the consumer would search for a high quality firm, as nothing is lost by doing so.

If both types of firms offer service, the cost of searching consists of the forgone service (recall that first period service at a new firm is zero). Assume searching for a high quality firm is not optimal, given that the best firm encountered so far is low quality and all firms offer service. In other words a consumer always returns to the first firm he encounters. Similar to a derivation as in equation (5.2), the expected value at $q = q_l$ is then $E_{\rho'} V^C(q_l, \rho') = u_0^\theta + \int_{u_0^\theta - q_l - \bar{s}}^{\bar{\rho}} [1 - F(\rho)]d\rho$. The condition under which returning to the low quality firm rather than searching is optimal is then

$$\begin{aligned} & (1 - \delta)(q_l + \bar{s} + \rho) + \delta E_{\rho'} V^C(q_l, \rho') \\ \geq & (1 - \delta)(E_{q|\gamma}(q) + \rho) + \delta E_{\tilde{q}|\gamma} E_{\rho'} \max\{V^C(q_l, \rho'), V^C(\tilde{q}, \rho')\}, \end{aligned}$$

or

$$(1 - \delta) (q_l - E_{q|\gamma}(q) + \bar{s}) \geq \delta\gamma [E_{\rho'} V^C(q_h, \rho') - E_{\rho'} V^C(q_l, \rho')].$$

Substitution and division by γ yields

$$(1 - \delta) \left(q_l - q_h + \frac{\bar{s}}{\gamma} \right) \geq \delta \int_{u_0^\theta - q_h - \bar{s}}^{u_0^\theta - q_l - \bar{s}} [1 - F(\rho)]d\rho. \quad (5.6)$$

Since $\int_{u_0^\theta - q_l - \bar{s}}^{u_0^\theta - q_h - \bar{s}} [1 - F(\rho)] d\rho > 0$ and independent of δ , and $\gamma > 0$, there exists δ^θ such that for $\delta > \delta^\theta$ condition (5.6) cannot hold, where δ^θ is defined as the survival probability that solves (5.6) with equality. For $\bar{s} < \gamma(q_h - q_l)$, $\delta^\theta \leq 0$. If $\delta > \underline{\delta} \equiv \max\{0, \delta^w, \delta^p\}$, all consumers will search for high quality firms. This establishes lemma 1. Note that for $\gamma \geq \lambda$ a bound $\underline{\delta}$ can be established independently of the exact value of γ by finding the fixed point of the equality in (5.6) when γ is replaced by λ . *Q.E.D.*

Proof of Lemma 4 : Consider the mapping $\tau : [\lambda, 1] \times [u_0^w - q_h - \bar{s}, u_0^p - q_l] \rightarrow [\lambda, 1] \times [u_0^w - q_h - \bar{s}, u_0^p - q_l]$ such that $\tau(\gamma, \hat{\rho}) = \begin{pmatrix} \tau_1(\gamma, \hat{\rho}_l) \\ \tau_2(\gamma, \hat{\rho}_l) \end{pmatrix}$. Similar to equation (3.10) let $\tau_1(\gamma, \hat{\rho}_l)$ be defined as

$$\tau_1(\gamma, \hat{\rho}_l^\theta) = 1 - \frac{[1 - \delta + \delta B^\theta] [1 - \lambda]}{1 - \delta + \delta(1 - \lambda + \gamma)B^\theta + \left[\delta\lambda + \frac{\delta^2}{1-\delta}B^\theta\gamma\right] C^\theta}, \quad (5.7)$$

with $B^\theta \equiv 1 - F(\hat{\rho}_l^\theta)$ and $C^\theta \equiv 1 - F(u_0^\theta - q_h - s)$. When the wealthy follow other wealthy consumers, $\theta = w$. However, the analysis holds similarly for the poor following other poor, i.e., $\theta = p$. For $\gamma \in [\lambda, 1]$ the multiplier of $(1 - \lambda)$ is strictly smaller than 1 and so $\tau_1(\gamma, \hat{\rho}_l^\theta) > \lambda$. Clearly $\tau_1(\gamma, \hat{\rho}_l^\theta) < 1$. Similar to equation (3.1), let $\tau_2(\gamma, \hat{\rho}_l^\theta) = \tau_2(\gamma)$ be implicitly defined by

$$\tau_2(\gamma) = u_0^\theta - E_{q|\gamma}(q) - \frac{\delta\gamma}{1-\delta} \int_{u_0^\theta - q_h - s_h(\theta)}^{\tau_2(\gamma)} [1 - F(\rho)] d\rho. \quad (5.8)$$

The function τ is continuous. For τ_1 this is easy to see. For τ_2 , note that in (5.8) γ as a function of τ_2 is continuous and strictly monotone. Therefore $\tau_2(\gamma)$ is also continuous. Domain and codomain of τ are identical, and they are compact subsets of \mathbb{R}^2 . By Brouwer's fixed point theorem there exists a fixed point of τ . *Q.E.D.*

Proof of Proposition 1: Consider $\theta \in \{p, w\}$. For $\gamma \in (\lambda, 1)$ we have $\gamma^\theta > \lambda$ (see discussion in proof of lemma 4, where $\tau_1(\gamma, \hat{\rho}_l^\theta)$ corresponds to γ^θ). To compare γ^w and γ^p consider the general form of (3.10) with w replaced by θ , where $\theta \in \{p, w\}$. Some algebra reveals that $(\partial\gamma^\theta/\partial B^\theta) > 0$ iff $(\gamma - \lambda)\delta(1 - \delta) + \delta^2(\gamma - \lambda)C^\theta > 0$, which holds since $\gamma \in (\lambda, 1)$. Clearly $(\partial\gamma^\theta/\partial C^\theta) > 0$. Therefore $\gamma^w > \gamma^p$ if $C^p < C^w$ and $B^p < B^w$, which is by lemma (3) the case for $s_h(w) \geq s_h(p)$ or $\bar{s} < u_0^p - u_0^w$. By the same lemma $s_h(p) - s_h(w) = \bar{s} > \xi_\gamma$ implies $C^p > C^w$ and $B^p > B^w$, which in turn implies $\gamma^w < \gamma^p$. In the intermediate case of $s_h(p) - s_h(w) = \bar{s} \in (u_0^p - u_0^w, \xi_\gamma)$ we have $C^p > C^w$ but $B^p < B^w$. If $s_h(p) - s_h(w) \approx u_0^p - u_0^w$, then $C^p \approx C^w$ but $B^p < B^w$ and therefore $\gamma^w > \gamma^p$. If $s_h(p) - s_h(w) \approx \xi_\gamma$, then $C^p > C^w$ but $B^p \approx B^w$ and therefore $\gamma^w < \gamma^p$. If $s_h(p) - s_h(w) = \bar{s} \in (u_0^p - u_0^w, \xi_\gamma)$ an increase in \bar{s} increases C^p and B^p but leaves C^w and B^w unchanged, and there exists unique $\hat{\xi}_\gamma \in (u_0^p - u_0^w, \xi_\gamma)$ for which $s_h(p) - s_h(w) = \hat{\xi}_\gamma$ implies $\gamma^w < \gamma^p$. *Q.E.D.*

Proof of Proposition 2: To illustrate how Π is calculated, consider the following candidate stationary equilibrium: All consumers follow the wealthy, the wealthy are promised service by high quality and not by low quality firms, and no firm promises service to the poor. In this case, the benefit of a wealthy consumer to a high quality firm, denoted Π^{wh} , comprises the wealthy consumer's own contribution $P - c$, plus the life-time contributions of his followers. The expected number N_w of wealthy followers in the next period is given by the number of consumers who are searching in that period divided by the number of all wealthy who are consuming, i.e., $N_w = \frac{\varphi_l^w}{\varphi_l^w + \varphi_h^w} = 1 - \gamma$. In subsequent periods they consume with probability $1 - F(u_0^w - q_h - \bar{s})$ conditional on surviving. They generate benefit Π^{wh} every time they visit. These followers do not get service on their first visit to the firm, and finally, there are $N_p = \frac{\varphi_l^p}{\varphi_l^w + \varphi_h^w} \frac{1-\alpha}{\alpha}$ poor consumers who follow in the next period. In every subsequent period they consume with probability $1 - F(u_0^p - q_h)$ if they survive. They generate benefit P each time they consume. Thus, the contribution of a wealthy consumer is given by

$$\begin{aligned} \Pi^{wh} = P - c &+ \beta N_w \Pi^{wh} \left[1 + \frac{\delta\beta}{1-\delta\beta} (1 - F(u_0^w - q_h - s)) \right] + \beta N_w c \quad (5.9) \\ &+ \beta N_p P \left[1 + \frac{\delta\beta}{1-\delta\beta} (1 - F(u_0^p - q_h)) \right]. \end{aligned}$$

The proof of the proposition is divided into three lemmata. The following lemma establishes that a leader's benefit to a firm can be arbitrarily high if he is followed by sufficiently many customers of the other type.

Lemma 5 *Fix $M > 0$. Assume type θ customers are being followed by consumers of the other type $\bar{\theta} \neq \theta$. Assume the type $\bar{\theta}$ consumers do not receive service. Then for any $\delta \in (0, 1)$ there exists $\bar{\alpha} > 0$, such that for all $\alpha^\theta \in (0, \bar{\alpha})$ the benefit Π of a type θ customer to a firm is greater than M (independent of the service strategies toward type θ consumers).*

Proof: Since the type $\bar{\theta}$ followers do not receive service by assumption, they will search for high quality firms. The value of next-period type $\bar{\theta}$ followers to any firm due to a visit by a leader is at least

$$(1 - \delta)\alpha^{\bar{\theta}}[1 - F(u_0^{\bar{\theta}} - q_l)]^2(1 - \lambda)\delta\frac{1}{\alpha^{\bar{\theta}}}P\beta, \quad (5.10)$$

where $\alpha^w = \alpha$ and $\alpha^p = 1 - \alpha$. In every period there will be $(1 - \delta)\alpha^{\bar{\theta}}$ newborn followers of type $\bar{\theta}$ who go into the market with probability greater than $[1 - F(u_0^{\bar{\theta}} - \underline{q})] > 0$, do not find a sufficiently good firm with probability $(1 - \lambda)$, survive another period with probability δ , and consume again with probability of at least $[1 - F(u_0^{\bar{\theta}} - q_l)]$. This time

they follow a leader who was in the market the previous period, of whom there are at most α^θ . They pay price P , and since they follow a period later than the visit of the leader, their value is discounted by β . The expression goes to infinity as α^θ going to zero.

The firm might incur service costs for the leader, but these are easily offset by his immediate type $\bar{\theta}$ followers. The leader might also have followers of his own type, which themselves bring a benefit larger than M in the period after and will therefore increase this consumers benefit even more. *Q.E.D.*

Recall that a high quality firm can induce a customer to return by promising service. The following lemma shows that a high quality firm will provide service when the customer's profit contribution is sufficiently large. Let $\Pi^{\theta h}$ denote the benefit of one-time consumption of a type θ consumer for a high quality firm.

Lemma 6 *There exists $M > 0$ such that in any stationary equilibrium with $\Pi^{\theta h} > M$, a high quality firm will promise service \bar{s} in any period to type θ consumers.*

Proof: Let s be the equilibrium strategy of a high quality firm to a type θ customer that generates the benefit of $\Pi^{\theta h}$ for the firm. Let $\check{s} \neq s$ be a one-shot deviation in the service promise.

If promising \check{s} instead of $s = \bar{s}$ results in the customer searching for another firm and never returning, then for $\Pi^{\theta h} > M = c$ offering the service is optimal, since by offering service the firm retains the business of this consumer and gains $\Pi^{\theta h} - c$ when he returns. As discussed above, our restrictive equilibrium concept necessitates the discussion of the case where the consumer would return even if service were not promised for one period.²⁵ For this case the proof is divided into two parts. The first establishes that increasing (decreasing) the service promise increases (decreases) the probability with which the consumer returns by a finite amount. The second provides the lower bound for the profitability of the consumer such that the threat of a potential time delay warrants service promises.

For the first part we discuss the consumer's reaction to a deviation. In equilibrium the customer returns whenever $\rho \geq \hat{\rho}_h^\theta = u_0^\theta - q_h - s$, otherwise he does not consume. Assume that the customer also chooses the firm rather than random sampling at $\check{s} \in \{0, \bar{s}\} \setminus \{s\}$. Let $V^C(q_h, \rho) = V^C(\rho)$ and V^F be the flow payoff of this strategy for the customer and the firm respectively. Consider the customer's response to a one-shot deviation by the firm. The value function $\check{V}^C(\rho)$ of the customer for the period directly

²⁵Since the equilibrium strategy of a firm is a function $s(\theta)$ independent of the history, after a one-shot deviation a consumer still expects to get the equilibrium service promise whenever he returns. This belief makes it hard to sustain an equilibrium at $s = \bar{s}$ and easy to sustain an equilibrium at $s = 0$, because the consumer expects the change for only a single period and reacts little (compared e.g. to the case where he expects the change to continue forever).

after the deviation is

$$\check{V}^C(\rho) = \max\{(1 - \delta)(q_h + \check{s} + \rho) + \delta EV^C(\rho), (1 - \delta)u_0^\theta + \delta E\check{V}^C(\rho)\}. \quad (5.11)$$

Let $\check{\rho}$ be the value for which the first term in the max operator is equal to the second term, i.e.,

$$(1 - \delta)(q_h + \check{s} + \check{\rho}) + \delta EV^C(\rho) = (1 - \delta)u_0^\theta + \delta E\check{V}^C(\rho). \quad (5.12)$$

This implies that the customer will return to the firm when $\rho \geq \check{\rho}$, and will not consume otherwise. Then

$$E\check{V}^C(\rho) = \int_{\check{\rho}}^{\bar{\rho}} [(1 - \delta)(q_h + \check{s} + \rho) + \delta EV^C(\rho)] f(\rho) d\rho + \int_{\underline{\rho}}^{\check{\rho}} [(1 - \delta)u_0^\theta + \delta E\check{V}^C(\rho)] f(\rho) d\rho.$$

Therefore

$$\begin{aligned} (1 - \delta F(\check{\rho}))E\check{V}^C(\rho) &= (1 - F(\check{\rho})) [(1 - \delta)(q_h + \check{s}) + \delta EV^C(\rho)] \\ &\quad + \int_{\check{\rho}}^{\bar{\rho}} (1 - \delta)\rho f(\rho) d\rho + F(\check{\rho})(1 - \delta)u_0^\theta. \end{aligned} \quad (5.13)$$

Substituting (5.13) into the equation (5.12), integration by parts and rearranging yields:

$$(1 - \delta)(\check{\rho} + q_h + \check{s}) - \delta \int_{\check{\rho}}^{\bar{\rho}} (1 - F(\rho)) d\rho - u_0^\theta + \delta EV^C(\rho) = 0.$$

The value of $EV^C(\rho)$ is given by lemma (2). Substitution leads to

$$(1 - \delta)(\check{\rho} + q_h + \check{s} - u_0^\theta) + \delta \int_{u_0^\theta - q_h - s}^{\check{\rho}} (1 - F(\rho)) d\rho = 0. \quad (5.14)$$

Equation (5.14) has a unique solution. It also reveals that for $s = 0$ and $\check{s} = \bar{s}$ we have $\check{\rho} < u_0^\theta - q_h$, which implies that the frequency of consumption is increased by the deviation. Let ζ_s be the probability of returning each period under the equilibrium strategy s , and let $\check{\zeta}_s$ be the probability of returning next period after a one-shot deviation in the service promise. Then $\check{\zeta}_0 - \zeta_0 = (1 - F(\check{\rho})) - (1 - F(u_0^\theta - q_h)) > 0$. On the other hand for $s = \bar{s}$ and $\check{s} = 0$ equation (5.14) reveals that $\check{\rho} < u_0^\theta - q_h - \bar{s}$, which implies that the deviation decreases the frequency of consumption. That is, $\check{\zeta}_{\bar{s}} - \zeta_{\bar{s}} \equiv (1 - F(\check{\rho})) - (1 - F(u_0^\theta - q_h - \bar{s})) < 0$. Hence, a change in service provision changes the frequency of consumption by a finite amount, i.e. $\Delta\zeta \equiv \min\{\check{\zeta}_0 - \zeta_0, |\check{\zeta}_{\bar{s}} - \zeta_{\bar{s}}|\} > 0$.

For the second part we discuss the firm's incentive to deviate. We show that for Π large enough $s = 0$ cannot be an equilibrium strategy since a one-shot deviation would be profitable. We also show that $s = \bar{s}$ is an equilibrium strategy.

Consider first the case where the candidate equilibrium strategy is $s = 0$, the one-shot deviation is $\check{s} = \bar{s}$. In this case $(\check{\zeta}_0 - \zeta_0) > 0$. Note that the effective discount factor for the firm in this case is $\delta_F = \delta\beta$ because the firm discounts with β and the survival probability of the customer is δ . Normalizing profits by $(1 - \delta_F)$, the equilibrium value to the firm is $V^F = \zeta_0 \Pi^{\theta h}$. The value to the firm from period $t + 1$ onward after a one-shot deviation in period t is

$$\check{V}^F = \check{\zeta}_0((1 - \delta_F)(\Pi^{\theta h} - c) + \delta_F V^F) + (1 - \check{\zeta}_0)\delta_F \check{V}^F.$$

A one-shot deviation is *profitable* if $\check{V}^F > V^F$, or equivalently $\Pi^{\theta h} > \frac{\check{\zeta}_0}{\zeta_0 - \check{\zeta}_0}c$. This is fulfilled if $M \geq \frac{1}{\Delta\zeta}c$.

Consider now the case of $s = \bar{s}$ and $\check{s} = 0$. In this case $(\check{\zeta}_{\bar{s}} - \zeta_{\bar{s}}) < 0$. The equilibrium (flow) value to the firm is $V^F = \zeta_{\bar{s}} \Pi^{\theta h}$. The flow value to the firm from period $t + 1$ onward after a one-shot deviation in period t is

$$\check{V}^F = \check{\zeta}_{\bar{s}}((1 - \delta_F)(\Pi^{\theta h} + c) + \delta_F V^F) + (1 - \check{\zeta}_{\bar{s}})\delta_F \check{V}^F.$$

A one-shot deviation is *not profitable* if $\check{V}^F \leq V^F$, or equivalently $\Pi^{\theta h} \geq \frac{\check{\zeta}_{\bar{s}}}{\zeta_{\bar{s}} - \check{\zeta}_{\bar{s}}}c$. This is fulfilled if $M \geq \frac{1}{\Delta\zeta}c$. Therefore for $\Pi^{\theta h} > M \geq \frac{1}{\Delta\zeta}c$ the only equilibrium strategies for high quality firms is $s = \bar{s}$. *Q.E.D.*

Finally we show that high quality firms will always outbid low quality firms:

Lemma 7 *Let $\delta \in (\underline{\delta}, 1)$. In any stationary equilibrium, either $s_h(\theta) \geq s_l(\theta)$ for $\theta \in \{p, w\}$, or $s_h(\theta) < s_l(\theta)$ but type θ consumers nevertheless do not return to low quality firms.*

Proof: Assume $s_h(\theta) < s_l(\theta)$ and type θ customers stop searching when they have found a low quality firm. There are two possibilities: Either they stop searching at the first firm they encounter, in which case $\gamma^\theta = \lambda$. Or they do not return to high quality firms but keep searching for a low quality firm. In this case $\gamma^\theta < \lambda$.

Call type θ consumers group Y, and type $\bar{\theta}$ consumers group Z. Group Y consumers must have some consumers that follow them or service would not be profitable. It cannot be that every consumer who is searching follows group Y, because group Z would then not receive service as it has no followers, and would therefore look for high quality firms. If $\gamma^\theta = \lambda$, by Proposition 1 $\gamma^{\bar{\theta}} > \lambda$ and group Z consumers should follow members of their own group. If $\gamma^\theta < \lambda$, group Z consumers are better off sampling on their own. Therefore either group Y members are only followed by group Y members, or they are only followed by group Z members.

In the second case, both groups would have to continue searching for low qualities (plus service) after finding a high quality firm. Assume group Y did not; then they

would stay at the first firm they patronize. But then group Z has no followers, thus receives no service and will look for high quality firms. But then it is not optimal for group Z to follow group Y. Assume group Z did not leave high quality firms; then they either stay at the first firm they encounter and do not follow group Y, or they only look for high quality firms, in which case following group Y is suboptimal. Therefore it must be that group Z searches for low quality firms, which implies that group Z also receives service from low quality firms. To receive service, it must be that they are followed by group Y. So both groups receive service from low quality firms and not from high quality firms. In this case, if the poor are leaving high quality firms to search for low quality plus service, then the wealthy strictly prefer to leave high quality firms to receive low quality plus service (as their higher frequency of consumption is similar to a lower discount factor). Yet by an argument similar to proposition 1, the signal of the wealthy is more informative about finding low quality firms when both types search for them and get identical service. Therefore the wealthy would not follow the poor, and this case cannot constitute an equilibrium.

We are therefore left with the case in which each group Y member is, in equilibrium, followed by some expected number N_θ of other members of its own group (and none of the other group). The candidate equilibrium profit contribution $\Pi^{\theta l}$ that a low quality firm receives from a group Y customer returning one more time is

$$\Pi^{\theta l} = P - c + \beta N_\theta \Pi^{\theta l} \left[1 + \frac{\delta \beta}{1 - \delta \beta} (1 - F(u_0^\theta - q_l - \bar{s})) \right] + \beta N_\theta c. \quad (5.15)$$

The derivation is similar to that of equation (5.9). In the stationary setting

$$\beta N_\theta \left[1 + \frac{\delta \beta}{1 - \delta \beta} (1 - F(u_0^\theta - q_l - \bar{s})) \right] < 1.$$

Solving for $\Pi^{\theta l}$ yields:

$$\Pi^{\theta l} = \frac{P - c + \beta N_\theta c}{1 - \beta N_\theta \left[1 + \frac{\delta \beta}{1 - \delta \beta} (1 - F(u_0^\theta - q_l - \bar{s})) \right]}. \quad (5.16)$$

For this to be an equilibrium, $\Pi^{\theta l} \geq 0$. Consider first the case where $\Pi^{\theta l} > 0$, i.e. $P - c + \beta N_\theta c > 0$. In this case high quality firms have an incentive to deviate and also offer service to group Y consumers, which upsets the equilibrium. To see this, note that for a high quality firm the candidate equilibrium profit contribution from a group Y consumer is zero after the first period of consumption, because he does not consume there again. Deviating and offering service to the customer and all his followers generates the profit contribution

$$\Pi' = P - c + \beta N_\theta \Pi' \left[1 + \frac{\delta \beta}{1 - \delta \beta} (1 - F') \right] + \beta N_\theta c,$$

where $1 - F'$ is the probability with which a customer that is offered service is returning. Since $\bar{\rho} > u_0^\theta - q_h - \bar{s}$, the frequency $1 - F' > 0$. Since $P - c + \delta N_\theta c > 0$, it follows that $\Pi' > 0$.²⁶ But then high quality firms would offer service.

Consider now the case $\Pi^{\theta l} = 0$, i.e. $P - c + \beta N_\theta c = 0$. Therefore, low quality firms are indifferent between promising service or not. In this case high quality firms are also indifferent between offering service or not. By the tie-breaking rule we employed, both types of firms offer service.²⁷ However, consumers then do not search for low quality firms; consequently high quality firms offer service, and group Y customers would not search for low qualities. *Q.E.D.*

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²⁶Since this is a deviation from a steady state, $\beta N_r \Pi' \left[1 + \frac{\delta \beta}{1 - \delta \beta} (1 - F') \right]$ might be larger than 1, in which case the discounted profit from offering service is unbounded.

²⁷This is the only place where we use this tie-breaking rule. The result holds also when we employ the assumption that firms do not offer service when indifferent. The point is that both types of firms resolve indifference the same way. Moreover, simple restrictions such as a high survival rate δ , a high cost-price wedge $c - P$ or a modest service influence \bar{s} would also guarantee the result, as they rule out indifference.

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